

# Can quintessence and phantom cause the late time acceleration of the Universe?

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## Abstract

In this Letter, we consider the Universe at the late stage of its evolution and deep inside the cell of uniformity. At these scales, the Universe is filled with inhomogeneously distributed discrete structures (galaxies, groups and clusters of galaxies). Supposing that the Universe contains also the cosmological constant and quintessence/phantom fluids with the constant equation of state parameter  $\omega$ , we investigate scalar perturbations of the FRW metrics due to inhomogeneities. Our analysis shows that, to be compatible with the theory of scalar perturbations, the quintessence/phantom, first, should be clustered and, second, should have the equation of state parameter  $\omega = -1/3$ . Therefore, this quintessence neither accelerates nor decelerates the Universe. We also obtain the equation for the nonrelativistic gravitational potential created by a system of inhomogeneities. Due to the quintessence, the physically reasonable solutions take place for flat, open and closed Universes. Quintessence is concentrated around the inhomogeneities and results in screening of the gravitational potential.

*Keywords:* quintessence, phantom, cosmological constant, late time acceleration, inhomogeneous Universe, scalar perturbations

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## 1. Introduction

The accelerated expansion of the Universe at late stages of its evolution, found little more than a decade ago [1, 2], is one of the most intriguing puzzles of modern physics and cosmology. Recognition of this fact was the awarding of the Nobel Prize in 2011 to Saul Perlmutter, Adam Riess and Brian Schmidt. After their discovery, there were numerous attempts to explain the nature of

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such acceleration. Unfortunately, there is no satisfactory explanation so far (see, e.g., the state of art in [3]). According to the recent observations [4, 5], the  $\Lambda$ CDM model is the preferable one. Here, the accelerated expansion is due to the cosmological constant. However, there is a number of problems associated with the cosmological constant. Maybe, one of the main of them consists in the adjustment mechanism which could compensate originally huge vacuum energy down to the cosmologically acceptable value and to solve the coincidence problem of close magnitudes of the non-compensated remnants of vacuum energy and the energy density of the Universe at the present time [6]. On the other hand, it is well known that perfect fluids with the equation of state parameters  $\omega < -1/3$  can also cause the accelerated expansion of the Universe. Such fluids are called quintessence [7, 8, 9] and phantom [10, 11] for  $-1 < \omega < 0$  and  $\omega < -1$ , respectively. Therefore, they can be an alternative to the cosmological constant. It is of interest to investigate the viability of these models.

In our Letter, we consider the compatibility of these models with scalar perturbations of the Friedmann-Robertson-Walker (FRW) metrics at late stages of the Universe evolution. We explore a simplified model with a constant equation of state parameter  $\omega = \text{const}$ . We demonstrate that, first, these fluids must be clustered (i.e. inhomogeneous) and, second,  $\omega = -1/3$  is the only parameter which is compatible with the theory of scalar perturbations. Therefore, this quintessence neither accelerates nor decelerates the Universe. We also obtain formulas for the nonrelativistic gravitational potential created by a system of inhomogeneities (galaxies, groups and clusters of galaxies). We show that due to the quintessence, the physically reasonable expressions take place for flat, open and closed Universes. If quintessence is absent, the hyperbolic space is preferred [12]. Hence, quintessence can play an important role.

The Letter is structured as follows. In Sec. 2, we consider scalar perturbations in the Friedmann Universe filled with the cosmological constant, pressureless dustlike matter (baryon and dark matter) and quintessence/phantom fluids. Here, we get the equation for the nonrelativistic gravitational potential. In Sec. 3, we find solutions of this equation for an arbitrary system of inhomogeneities for flat, open and closed Universes. These solutions have the Newtonian limit in the vicinity of inhomogeneities and are finite at any point outside inhomogeneities. The main results are summarized in concluding Sec. 4.

## 2. Scalar perturbations of FRW Universe

### *Homogeneous background.*

To start with, we consider a homogeneous isotropic Universe described by the FRW metrics

$$ds^2 = a^2 (d\eta^2 - \gamma_{\alpha\beta} dx^\alpha dx^\beta) = a^2 (d\eta^2 - d\chi^2 - \Sigma^2(\chi) d\Omega_2^2) , \quad (1)$$

where

$$\Sigma(\chi) = \begin{cases} \sin \chi, & \chi \in [0, \pi] & \text{for } \mathcal{K} = +1 \\ \chi, & \chi \in [0, +\infty) & \text{for } \mathcal{K} = 0 \\ \sinh \chi, & \chi \in [0, +\infty) & \text{for } \mathcal{K} = -1 \end{cases} \quad (2)$$

and  $\mathcal{K} = -1, 0, +1$  for open, flat and closed Universes, respectively. As matter sources, we consider the cosmological constant  $\Lambda$ , pressureless dustlike matter (in accordance with the current observations [4, 5], we assume that dark matter (DM) gives the main contribution to this matter) and an additional perfect fluid with the equation of state  $\bar{p} = \omega \bar{\varepsilon}$  where  $-1 < \omega < 0$  for the quintessence and  $\omega < -1$  for the phantom fluid. In the present Letter,  $\omega = \text{const}$ . The overline denotes homogeneous quintessence/phantom fluids. It can be easily seen from the conservation equation that in the case of the homogeneous perfect fluid

$$\bar{\varepsilon} = \varepsilon_0 \frac{a_0^{3(1+\omega)}}{a^{3(1+\omega)}}, \quad (3)$$

where  $a_0$  is the scale factor at the present time and  $\varepsilon_0$  is the current value of the energy density  $\bar{\varepsilon}$ .

Because we consider the late stages of the Universe evolution, we neglect the contribution of radiation. Therefore, the Friedmann equations read

$$\frac{3(\mathcal{H}^2 + \mathcal{K})}{a^2} = \kappa \bar{T}_0^0 + \Lambda + \kappa \bar{\varepsilon} \quad (4)$$

and

$$\frac{2\mathcal{H}' + \mathcal{H}^2 + \mathcal{K}}{a^2} = \Lambda - \kappa \omega \bar{\varepsilon}, \quad (5)$$

where  $\mathcal{H} \equiv a'/a \equiv (da/d\eta)/a$  and  $\kappa \equiv 8\pi G_N/c^4$  ( $c$  is the speed of light and  $G_N$  is the Newton's gravitational constant). Here,  $\bar{T}^{ik}$  is the energy-momentum tensor of the average pressureless dustlike matter. For such matter, the energy density  $\bar{T}_0^0 = \bar{\rho}c^2/a^3$  is the only nonzero component.  $\bar{\rho} = \text{const}$  is the average rest mass density [12]. It is worth noting that in the case  $\mathcal{K} = 0$  the comoving coordinates  $x^\alpha$  may have a dimension of length, but then the conformal factor  $a$  is dimensionless, and vice versa. However, in the cases  $\mathcal{K} = \pm 1$  the dimension of  $a$  is fixed. Here,  $a$  has a dimension of length and  $x^\alpha$  are dimensionless. For consistency, we shall follow this definition for  $\mathcal{K} = 0$  as well. For such choice of the dimension of  $a$ , the rest mass density has a dimension of mass.

It is well known that quintessence can provide the accelerated expansion of the Universe if the equation of state parameter satisfies the condition  $\omega < -1/3$ . We can easily see it if we rewrite equations (4) and (5) as follows:

$$\frac{\ddot{a}}{a} = -\frac{\kappa \bar{\rho} c^4}{6a^3} + \frac{\Lambda c^2}{3} - \frac{\kappa c^2}{6} \frac{\varepsilon_0 a_0^{3(1+\omega)}}{a^{3(1+\omega)}} (1 + 3\omega), \quad (6)$$

where overdots denote the differentiation with respect to synchronous time  $t$  connected with conformal time  $\eta$ :  $cdt = ad\eta$ .

*Scalar perturbations.*

Obviously, the inhomogeneities in the Universe result in scalar perturbations of the metrics (1). In the conformal Newtonian gauge, such perturbed metrics is [13, 14]

$$ds^2 \approx a^2 [(1 + 2\Phi)d\eta^2 - (1 - 2\Psi)\gamma_{\alpha\beta}dx^\alpha dx^\beta] , \quad (7)$$

where scalar perturbations  $\Phi, \Psi \ll 1$ . Following the standard argumentation, we can put  $\Phi = \Psi$ . We consider the Universe at the late stage of its evolution when the peculiar velocities of inhomogeneities (both for dustlike matter and quintessence/phantom fluids) are much less than the speed of light:

$$\frac{dx^\alpha}{d\eta} = a \frac{dx^\alpha}{dt} \frac{1}{c} \equiv \frac{v^\alpha}{c} \ll 1 . \quad (8)$$

We should stress that smallness of the nonrelativistic gravitational potential  $\Phi$  and peculiar velocities  $v^\alpha$  are two independent conditions (e.g., for very light relativistic masses the gravitational potential can still remain small). Under these conditions, the gravitational potential  $\Phi$  satisfies the following system of equations (see [12] for details):

$$\Delta\Phi - 3\mathcal{H}(\Phi' + \mathcal{H}\Phi) + 3\mathcal{K}\Phi = \frac{1}{2}\kappa a^2 \delta T_0^0 + \frac{1}{2}\kappa a^2 \delta\varepsilon , \quad (9)$$

$$\frac{\partial}{\partial x^\beta}(\Phi' + \mathcal{H}\Phi) = 0 , \quad (10)$$

$$\Phi'' + 3\mathcal{H}\Phi' + (2\mathcal{H}' + \mathcal{H}^2)\Phi - \mathcal{K}\Phi = \frac{1}{2}\kappa a^2 \delta p , \quad (11)$$

where the Laplace operator

$$\Delta = \frac{1}{\sqrt{\gamma}} \frac{\partial}{\partial x^\alpha} \left( \sqrt{\gamma} \gamma^{\alpha\beta} \frac{\partial}{\partial x^\beta} \right) \quad (12)$$

and  $\gamma$  is the determinant of  $\gamma_{\alpha\beta}$ . Following the reasoning of [12], we took into account that peculiar velocities of inhomogeneities are nonrelativistic and the contribution of  $\delta T_\beta^0$  is negligible compared to that of  $\delta T_0^0$  both for dustlike matter and quintessence/phantom. In other words, account of  $\delta T_\beta^0$  is beyond the accuracy of the model. This approach is completely consistent with [15] where it is shown that the nonrelativistic gravitational potential is defined by the positions of the inhomogeneities but not by their velocities (see Eq. (106.11) in this book). In the case of an arbitrary number of dimensions, a similar result was obtained in [16]. On the other hand, the motion of nonrelativistic inhomogeneities is defined by the gravitational potential (see, e.g., [17]). The perturbed DM remains nonrelativistic (pressureless) that results in the condition  $\delta T_\beta^\alpha = 0$ . For the quintessence/phantom we have  $\delta T_\beta^\alpha = -\delta p \delta_\beta^\alpha$ . In Eq. (9),  $\delta\varepsilon$  is a fluctuation of the energy density for quintessence/phantom, while  $\delta T_0^0$  is related to the fluctuation of the energy density of dustlike matter and has the form [12]:

$$\delta T_0^0 = \frac{\delta\rho c^2}{a^3} + \frac{3\bar{\rho}c^2\Phi}{a^3} , \quad (13)$$

where  $\delta\rho$  is the difference between real and average rest mass densities:

$$\delta\rho = \rho - \bar{\rho}. \quad (14)$$

From Eq. (10) we get

$$\Phi(\eta, \mathbf{r}) = \frac{\varphi(\mathbf{r})}{c^2 a(\eta)}, \quad (15)$$

where  $\varphi(\mathbf{r})$  is a function of all spatial coordinates and we have introduced  $c^2$  in the denominator for convenience. Below, we shall see that  $\varphi(\mathbf{r}) \sim 1/r$  in the vicinity of an inhomogeneity, and the nonrelativistic gravitational potential  $\Phi(\eta, \mathbf{r}) \sim 1/(ar) = 1/R$ , where  $R = ar$  is the physical distance. Hence,  $\Phi$  has the correct Newtonian limit near the inhomogeneities. Substituting the expression (15) into Eqs. (9) and (11), we get the following system of equations:

$$\frac{1}{a^3} (\Delta\varphi + 3\mathcal{K}\varphi) = \frac{1}{2}\kappa c^2 \delta T_0^0 + \frac{1}{2}\kappa c^2 \delta\varepsilon, \quad (16)$$

$$\frac{1}{a^3} (\mathcal{H}' - \mathcal{H}^2 - \mathcal{K})\varphi = \frac{1}{2}\kappa c^2 \delta p. \quad (17)$$

From the Friedmann equations (4) and (5) we obtain

$$\frac{1}{a^3} (\mathcal{H}' - \mathcal{H}^2 - \mathcal{K}) = \frac{1}{2a} \left( -\kappa \bar{T}_0^0 - \kappa(1 + \omega)\bar{\varepsilon} \right). \quad (18)$$

Then, Eq. (17) reads

$$\left( -\kappa \frac{\bar{\rho} c^2}{a^4} - \kappa(1 + \omega) \frac{\varepsilon_0}{a_0} \frac{a_0^{4+3\omega}}{a^{4+3\omega}} \right) \varphi = \kappa c^2 \omega \delta\varepsilon. \quad (19)$$

It should be noted that we consider quintessence/phantom fluids without thermal coupling to any other type of matter. It means, in particular, that evolution of its homogeneous background as well as scalar perturbations occurs adiabatically or, in other words, without change of entropy. Therefore, we preserve the same linear equation of state  $\delta p = \omega \delta\varepsilon$  with the same constant parameter  $\omega$  for the scalar perturbations  $\delta p$  and  $\delta\varepsilon$  of pressure and energy density respectively, as for their background values  $\bar{p}$  and  $\bar{\varepsilon}$ .

Taking into account the expression (15), we get that in the right hand side of Eq. (13) the second term is proportional to  $1/a^4$  and should be dropped because we consider the nonrelativistic matter. This is the accuracy of our approach, i.e. for the terms of the form of  $1/a^n$ , we drop ones with  $n \geq 4$  and leave terms with  $n < 4$ . Obviously,  $4 + 3\omega < 4$  for  $\omega < 0$ . Hence, we can draw the important conclusion regarding the purely homogeneous non-clustered quintessence/phantom fluids with  $\delta p, \delta\varepsilon = 0$ . For these fluids, we arrive at a contradiction because in Eq. (19) the right hand side is equal to zero while the left hand side is nonzero. Therefore, such fluids are forbidden. Quintessence and phantom should be capable of clustering. In the papers [9, 18], it was also

pointed out that the quintessence has to be inhomogeneous. For inhomogeneous quintessence/phantom we get from Eq. (19) that

$$\delta\varepsilon = -\frac{1+\omega}{c^2\omega}\varepsilon_0\frac{a_0^{3+3\omega}}{a^{4+3\omega}}\varphi. \quad (20)$$

It is worth noting that in our model neither the nonrelativistic gravitational potential  $\Phi \sim 1/a$  nor the quintessence/phantom density contrast  $\delta\varepsilon/\bar{\varepsilon} \sim 1/a$  diverge with time (with the scale factor  $a$ ) in spite of the negative sign of the ratio  $\delta p/\delta\varepsilon$  which is often treated as the speed of sound squared. Substituting (20) into (16), we obtain within our accuracy

$$\begin{aligned} \frac{1}{a^3}(\Delta\varphi + 3\mathcal{K}\varphi) &= \frac{1}{2}\kappa c^2\frac{\delta\rho c^2}{a^3} - \frac{1}{2}\kappa c^2\frac{1+\omega}{c^2\omega}\varepsilon_0\frac{a_0^{3+3\omega}}{a^{4+3\omega}}\varphi \\ \Rightarrow \Delta\varphi + 3\mathcal{K}\varphi &= \frac{1}{2}\kappa c^4\delta\rho - \frac{1+\omega}{2\omega}\kappa\varepsilon_0 a_0^2\frac{a_0^{1+3\omega}}{a^{1+3\omega}}\varphi. \end{aligned} \quad (21)$$

In this equation, all terms except the last one do not depend on time. Therefore,  $\omega = -1/3$  is the only possibility to avoid this problem. Hence, we arrive at the following important conclusion. At the late stage of the Universe evolution, quintessence/phantom fluids are compatible with the scalar perturbations only if, first, they are inhomogeneous, and, second, they have the equation of state parameter  $\omega = -1/3$ . Such fluid is quintessence which cannot provide the late time acceleration of the Universe (i.e. this quintessence neither accelerates nor decelerates the Universe (see Eq. (6))).

For  $\omega = -1/3$ , the equation for the gravitational potential and the fluctuation of the energy density of the quintessence read, respectively:

$$\Delta\varphi + \left(3\mathcal{K} - \frac{8\pi G_N}{c^4}\varepsilon_0 a_0^2\right)\varphi = 4\pi G_N(\rho - \bar{\rho}) \quad (22)$$

and

$$\delta\varepsilon = \frac{2\varepsilon_0 a_0^2}{c^2 a^3}\varphi. \quad (23)$$

In the next section, we shall investigate Eq. (22) depending on the curvature parameter  $\mathcal{K}$ . We shall show that reasonable expressions of the conformal gravitational potential  $\varphi$  exist for any sign of  $\mathcal{K}$ . This takes place due to the presence of the quintessence with  $\omega = -1/3$ . If quintessence is absent, the hyperbolic model  $\mathcal{K} = -1$  is preferred [12]. Therefore, the positive role of quintessence is that its presence gives a possibility to consider models for any  $\mathcal{K}$ .

### 3. Gravitational potentials

It is convenient to rewrite Eq. (22) as follows:

$$\Delta\phi - \lambda^2\phi = 4\pi G_N\rho, \quad (24)$$

where the truncated gravitational potential is

$$\phi = \varphi - \frac{4\pi G_N \bar{\rho}}{\lambda^2}, \quad \lambda \neq 0, \quad (25)$$

and

$$\lambda^2 \equiv \frac{8\pi G_N}{c^4} \varepsilon_0 a_0^2 - 3\mathcal{K}. \quad (26)$$

On scales smaller than the cell of uniformity size (which is of the order of 150 Mpc) and on late stages of evolution, the Universe is filled with inhomogeneously distributed discrete structures (galaxies, groups and clusters of galaxies) with dark matter concentrated around these structures. Then, the rest mass density  $\rho$  reads [12]

$$\rho = \frac{1}{\sqrt{\gamma}} \sum_i m_i \delta(\mathbf{r} - \mathbf{r}_i), \quad (27)$$

where  $m_i$  is the mass of  $i$ -th inhomogeneity. Therefore, Eq. (24) satisfies the very important principle of superposition. It is sufficient to solve this equation for one gravitating mass  $m_i$  and obtain its gravitational potential  $\phi_i$ . The gravitational potential for all system of inhomogeneities is equal to a sum of potentials  $\phi_i$ . It is worth recalling that the operator  $\Delta$  is defined by Eq. (12). As boundary conditions, we demand that, first, the gravitational potential of a gravitating mass should have the Newtonian limit near this inhomogeneity  $\phi_i \sim 1/r$  and, second, this potential should converge at any point of the Universe (except the gravitating mass position).

It seems reasonable to assume also that the total gravitational potential averaged over the whole Universe is equal to zero (see, e.g., [12]):

$$\bar{\varphi} = \bar{\phi} + \frac{4\pi G_N \bar{\rho}}{\lambda^2} = 0, \quad \bar{\phi} = \sum_i \frac{1}{V} \int_V \phi_i dV, \quad (28)$$

where  $V$  is the volume of the Universe. This demand results in another physically reasonable condition:  $\bar{\delta\varepsilon} = 0$  (see Eq. (23)).

*Flat space:*  $\mathcal{K} = 0$ .

In the case  $\varepsilon_0 > 0 \rightarrow \lambda^2 = \frac{8\pi G_N}{c^4} \varepsilon_0 a_0^2 > 0$ , the solution of (24) for a separate mass  $m_i$  satisfying the mentioned above boundary conditions reads

$$\phi_i = -\frac{G_N m_i}{r} \exp(-\lambda r), \quad \lambda > 0, \quad 0 < r < +\infty. \quad (29)$$

It can be easily seen that this truncated potential has the Newtonian limit for  $r \rightarrow 0$ . This expression shows that quintessence results in the screening of the Newtonian potential. A similar effect for the Coulomb potential takes place in plasma. In our case, the screening originates due to specific nature of quintessence. It is worth mentioning that the exponential screening of the gravitational potential was introduced "by hand" in a number of models to solve

the famous Seeliger paradox (see, e.g., the review [19]). In our model, we resolve this paradox in a natural way due to the presence of quintessence.

For a many-particle system, the total gravitational potential takes the form

$$\varphi = -G_N \sum_i \frac{m_i}{|\mathbf{r} - \mathbf{r}_i|} \exp(-\lambda|\mathbf{r} - \mathbf{r}_i|) + \frac{4\pi G_N \bar{\rho}}{\lambda^2}. \quad (30)$$

Substituting (30) into (23), we get for the fluctuation of the quintessence energy density the following expression:

$$\delta\varepsilon = \frac{2\varepsilon_0 a_0^2}{c^2 a^3} \left( -G_N \sum_i \frac{m_i}{|\mathbf{r} - \mathbf{r}_i|} \exp(-\lambda|\mathbf{r} - \mathbf{r}_i|) + \frac{c^4 \bar{\rho}}{2\varepsilon_0 a_0^2} \right). \quad (31)$$

Therefore, we arrive at a physically reasonable conclusion that these fluctuations are concentrated around the matter/dark matter inhomogeneities and the corresponding profile is given by Eq. (31).

The averaged value of the  $i$ -th component of the truncated potential over some finite volume  $V_0$  is

$$\begin{aligned} \bar{\phi}_i &= \frac{4\pi}{V_0} \int_0^{r_0} \left[ -G_N m_i \frac{\exp(-\lambda r)}{r} \right] r^2 dr \\ &= -\frac{4\pi G_N m_i}{V_0} \left[ -\frac{\exp(-\lambda r_0)}{\lambda} \left( r_0 + \frac{1}{\lambda} \right) + \frac{1}{\lambda^2} \right]. \end{aligned} \quad (32)$$

Then, letting the volume go to infinity ( $r_0 \rightarrow +\infty \Rightarrow V_0 \rightarrow +\infty$ ) and taking all gravitating masses, we obtain

$$\bar{\phi} = -G_N \bar{\rho} \frac{4\pi}{\lambda^2}, \quad (33)$$

where  $\bar{\rho} = \lim_{V_0 \rightarrow +\infty} \sum_i m_i / V_0$ . Therefore, the averaged gravitational potential (28) is equal to zero:  $\bar{\varphi} = 0$ . Consequently,  $\bar{\delta\varepsilon} = 0$ .

The case  $\varepsilon_0 < 0 \Rightarrow \lambda^2 \equiv -\mu^2 < 0$  is not of interest. Here, we get the expression  $\phi_i = -(G_N m_i / r) \cos(\mu r)$  which does not have clear physical sense. Additionally, this expression does not allow the procedure of averaging.

*Spherical space:*  $\mathcal{K} = +1$ .

Let us consider, first, the case  $\lambda^2 = \frac{8\pi G_N}{c^4} \varepsilon_0 a_0^2 - 3 \equiv -\mu^2 < 0$ . This case is of interest because it allows us to perform the transition to small values of the energy density of quintessence:  $\varepsilon_0 \rightarrow 0$ . Here, the solution of (24) for a separate mass  $m_i$  is

$$\phi_i = -G_N m_i \frac{\sin \left[ (\pi - \chi) \sqrt{\mu^2 + 1} \right]}{\sin \left( \pi \sqrt{\mu^2 + 1} \right) \sin \chi}, \quad 0 < \chi \leq \pi. \quad (34)$$



For  $\sqrt{\mu^2 + 1} \neq 2, 3, \dots$  (we would remind that  $\mu^2 \neq 0$ ), this formula is finite at any point  $\chi \in (0, \pi]$  and has the Newtonian limit for  $\chi \rightarrow 0$ . In the case of absence of quintessence  $\varepsilon_0 = 0 \rightarrow \sqrt{\mu^2 + 1} = 2$ , this expression is divergent at  $\chi = \pi$ . We demonstrated this fact in our paper [12]. Therefore, quintessence gives a possibility to avoid this problem for the models with  $\mathcal{K} = +1$ . It can be easily verified that for the total system of gravitating masses, the averaged value of the total truncated potential has the form of (33) that results in  $\overline{\varphi} = 0 \Rightarrow \overline{\delta\varepsilon} = 0$ .

In the case  $\lambda^2 > 0$ , the formulas can be easily found from (34) with the help of analytical continuation  $\mu \rightarrow i\mu$ . In other words, it is sufficient in Eq. (34) to replace  $\mu^2$  by  $-\lambda^2$ . The obtained expression is finite for all  $\chi \in (0, \pi]$  and the averaged gravitational potential is equal to zero:  $\overline{\varphi} = 0 \Rightarrow \overline{\delta\varepsilon} = 0$ .

*Hyperbolic space:  $\mathcal{K} = -1$ .*

Here, the most interesting case corresponds to  $\lambda^2 = \frac{8\pi G_N}{c^4} \varepsilon_0 a_0^2 + 3 > 0$ . This choice of sign gives a possibility to perform the transition to small values of the energy density of quintessence:  $\varepsilon_0 \rightarrow 0$ . Then, the desired solution of Eq. (24) for a mass  $m_i$  is

$$\phi_i = -\frac{G_N m_i}{\sinh \chi} \exp\left(-\chi \sqrt{\lambda^2 + 1}\right), \quad 0 < \chi < +\infty. \quad (35)$$

If quintessence is absent ( $\varepsilon_0 = 0$ ), then we reproduce the formula obtained in [12]. On the other hand, the expression (35) shows that for  $\varepsilon_0 > 0 \rightarrow \lambda^2 + 1 > 4$ , quintessence enhances the screening of the gravitating mass. For a many-particle system, the total gravitational potential takes the form

$$\varphi = -G_N \sum_i m_i \frac{\exp(-l_i \sqrt{\lambda^2 + 1})}{\sinh l_i} + \frac{4\pi G_N \bar{\rho}}{\lambda^2}, \quad (36)$$

where  $l_i$  denotes the geodesic distance between the  $i$ -th mass  $m_i$  and the point of observation. Similarly, using Eq. (34), we can write the expression for the total potential in the case of the spherical space.

Taking into account that the averaged total truncated potential has again the form (33), the procedure of averaging leads to the physically reasonable result:  $\overline{\varphi} = 0 \Rightarrow \overline{\delta\varepsilon} = 0$ .

Concerning the case  $\lambda^2 < 0$ , the truncated gravitational potential is finite in the limit  $\chi \rightarrow +\infty$ . However, the procedure of averaging does not exist here. Therefore, this case is not of interest for us.

To conclude this section, we discuss briefly the case  $\lambda^2 = 0$ . For  $\mathcal{K} = 0, -1$ , the principle of superposition is absent now. To make the gravitational potential finite at any point including the spatial infinity, we need to cutoff it smoothly at some distances from each gravitating mass. If  $\mathcal{K} = 0$ , then quintessence is absent and this case was described in detail in [12]. It was shown that the averaged gravitational potential is not equal to zero. This is a disadvantage of such

models. In the case  $\mathcal{K} = +1$ , the principle of superposition can be introduced due to the finiteness of the total volume of the Universe. Here, the comoving averaged rest mass density can be split as follows:  $\bar{\rho} = \sum_i m_i / (2\pi^2) \equiv \sum_i \bar{\rho}_i$ . Then, Eq. (22) can be solved separately for each combination  $(m_i, \bar{\rho}_i)$ . As a result, the gravitational potential of the  $i$ -th mass is

$$\varphi_i = \frac{G_N m_i}{2\pi} - G_N m_i \frac{\cos \chi}{\sin \chi} \left(1 - \frac{\chi}{\pi}\right), \quad 0 < \chi \leq \pi. \quad (37)$$

This potential is convergent at any point  $\chi \neq 0$ , including  $\chi = \pi$ . It is not difficult to see that  $\bar{\varphi}_i = 0$ . Therefore, the total averaged gravitational potential is also equal to zero:  $\bar{\varphi} = \sum_i \bar{\varphi}_i = 0 \Rightarrow \bar{\delta\varepsilon} = 0$ .

#### 4. Conclusion

In our Letter, we have investigated the role of quintessence and phantom field for the Universe at late stages of its evolution. It is well known that these fields can be an alternative to the cosmological constant explaining the late time acceleration of the Universe. It happens if their parameter of the equation of state  $\omega < -1/3$ . To check the compatibility of these fluids with observations, we consider our Universe at scales much less than the cell of homogeneity size which is approximately 150 Mpc. At such distances, our Universe is highly inhomogeneous and the averaged Friedmann approach does not work here. We need to take into account the inhomogeneities in the form of galaxies, groups and clusters of galaxies. All of them perturb the FRW metrics.

To clarify the role of quintessence and phantom, we endowed the Universe with these fluids with a constant equation of state parameter  $\omega$ . We have shown that quintessence and phantom are compatible with the theory of scalar perturbations if they satisfy two conditions. First, these fluids must be clustered (i.e. inhomogeneous). Second, the parameter of the equation of state  $\omega$  should be  $-1/3$ . Therefore, this quintessence neither accelerates nor decelerates the Universe.

Then, we have obtained the equation for the nonrelativistic gravitational potential. We have shown that due to quintessence the physically reasonable solutions take place for flat, open and closed Universes. The presence of quintessence helps to resolve the Seeliger paradox [19] for any sign of the spatial curvature parameter  $\mathcal{K}$ . If quintessence is absent, the hyperbolic space is preferred [12]. Hence, quintessence can play an important role. Quintessence is concentrated around the inhomogeneities and results in screening of the gravitational potential.

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## References

- [1] A.G. Riess et al., Observational evidence from supernovae for an accelerating Universe and a cosmological constant, *Astron. J.* **116** (1998) 1009; (arXiv:astro-ph/9805201).
- [2] S. Perlmutter et al., Measurements of Omega and Lambda from 42 high-redshift supernovae, *Astrophys. J.* **517** (1999) 565; (arXiv:astro-ph/9812133).
- [3] M. Trodden, Dark Energy and cosmology; (arXiv:astro-ph/1212.6399).
- [4] E. Komatsu et al., Seven-year Wilkinson Microwave Anisotropy Probe (WMAP) observations: cosmological interpretation, *Astrophys. J. Suppl.* **192** (2011) 18; (arXiv:astro-ph/1001.4538).
- [5] G. Hinshaw et al., Nine-year Wilkinson Microwave Anisotropy Probe (WMAP) observations: cosmological parameter results; (arXiv:astro-ph/1212.5226).
- [6] A.D. Dolgov, Cosmic antigravity; (arXiv:astro-ph/1206.3725).
- [7] P.G. Ferreira and M. Joyce, Structure formation with a self-tuning scalar field, *Phys. Rev. Lett.* **79** (1997) 4740; (arXiv:astro-ph/9707286).
- [8] L. Wang and P.J. Steinhardt, Cluster abundance constraints for cosmological models with a timevarying, spatially inhomogeneous energy component with negative pressure, *Astrophys. J.* **508** (1998) 483; (arXiv:astro-ph/9804015).
- [9] I. Zlatev, L. Wang and P.J. Steinhardt, Quintessence, cosmic coincidence, and the cosmological constant, *Phys. Rev. Lett.* **82** (1999) 896; (arXiv:astro-ph/9807002).
- [10] R.R. Caldwell, A phantom menace? Cosmological consequences of a dark energy component with super-negative equation of state, *Phys. Lett. B* **545** (2002) 23; (arXiv:astro-ph/9908168).
- [11] S.M. Carroll, M. Hoffman and M. Trodden, Can the dark energy equation-of-state parameter  $w$  be less than -1? *Phys. Rev. D.* **68** (2003) 023509; (arXiv:astro-ph/0301273).
- [12] M. Eingorn and A. Zhuk, Hubble flows and gravitational potentials in observable Universe, *JCAP* **09** (2012) 026; (arXiv:astro-ph/1205.2384).
- [13] V.F. Mukhanov, H.A. Feldman and R.H. Brandenberger, Theory of cosmological perturbations, *Physics Reports* **215** (1992) 203.
- [14] D.S. Gorbunov and V.A. Rubakov, Introduction to the theory of the early Universe: cosmological perturbations and inflationary theory, World Scientific, Singapore, 2011.

- [15] L.D. Landau and E.M. Lifshitz, The classical theory of fields, fourth edition: volume 2 (course of theoretical physics series), Oxford Pergamon Press, Oxford, 2000.
- [16] M. Eingorn and A. Zhuk, Classical tests of multidimensional gravity: negative result, *Class. Quant. Grav.* **27** (2010) 205014 (arXiv:gr-qc/1003.5690).
- [17] M. Eingorn, A. Kudinova and A. Zhuk, Dynamics of astrophysical objects against the cosmological background; (arXiv:astro-ph/1211.4045).
- [18] P.J. Steinhardt, L. Wang and I. Zlatev, Cosmological tracking solutions, *Phys. Rev. D* **59** (1999) 123504; (arXiv:astro-ph/9812313).
- [19] J.D. Norton, The cosmological woes of Newtonian gravitation theory, in H. Goenner, J. Renn, J. Ritter and T. Sauer, eds., *The expanding worlds of General Relativity: Einstein studies*, volume 7, Boston: Birkhauser, pp. 271-323, 1999.